Modeling of Sound Power Transmission within Duct Systems

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Abstract

External boundaries of acoustic devices can channel sound propagation, and in some cases can create buildup or attenuation of acoustic energy within a confined space. In this paper, it is proposed an efficient practical numerical method (based on FEM) of calculation of attenuation of sound power transmission through ducts. The method shows its viability by presenting the reasonably consistent anticipation of the experimental result. One can observe the mechanical behaviour of the duct's medium for lower frequencies (high transmission loss) and wave behaviour for higher frequencies (small or zero attenuation). The authors proved that mechanical vibrations of medium reduce the possibility of acoustic energy transmission in duct systems. The radiation impedance for the duct is calculated as well.

Keywords: transmission loss, finite element method, dimensionless acoustic equations, sound power

1. Introduction

Acoustics is the physics of sound. Sound results when the air is disturbed by some source. Sound is the sensation of very small rapid changes in the air pressure above and below a static value (atmospheric pressure). Associated with a sound pressure wave is a flow of energy. Physically, sound in air is a longitudinal wave where the wave motion is in the direction of the movement of energy.

Plane waves of constant frequency propagating through bulk materials have the complex acoustic pressure amplitudes that typically decrease exponentially with increasing propagation distance. A lot of devices with duct systems like air-conditioning, heating equipment, frig, and so on are being used in people's life. External boundaries of acoustic devices can channel sound propagation, and in some cases can create buildup or attenuation of acoustic energy within a confined space. Ducts act as guides of acoustic waves, and the net flow of energy. The general theory of guided waves applies and leads to representation in terms of guided modes [1].

The study of the diffraction, reflection and transmission coefficients of acoustic waves in ducts of constants and continuously varying cross-sectional area has been of interest to many researchers in the past (e.g. [2-3]). A method of solution based on a variational principle has been presented in the paper [2] for the acoustic wave propagation in ducts having a continuous change in the cross-sectional area. Wilson and Soroka [3] presented an approximate solution for the diffraction of a plane's sound wave incident normally on a circular aperture in a plane rigid wall of finite thickness.

On this consideration, in this paper, it is proposed an efficient practical numerical method for calculation of attenuation of the sound of ducts and the whole acoustic structure based on the Finite Element Method (FEM). The transmission loss estimation by the proposed numerical method was tested by comparison with the experimental one on a sound attenuation in a hard-walled pipe [4]. The method shows its viability by presenting the reasonably consistent anticipation of the experimental result. In this paper, the method is applied to a cylindrical duct with a theoretical plane wave on inlet and outlet opened to free space. Authors have calculated radiation impedance of outlet of duct, as well. Proper radiation impedance outlet boundary condition is very important in correct calculation of the transmission loss.

2. Sound power transmission

Considered a cylindrical duct with rigid walls links source area and free space (Figure 1). Acoustic power is delivered to the inlet of the duct by plane wave (W_{in}) and radiated from the outlet of the duct into free space (W_{out}). The wave which is radiated from the cylindrical duct is not plane wave due to the wave's diffraction on the interface between the duct and free space. In this paper free space is modeled by the cylindrical domain with at least ten times greater radius.

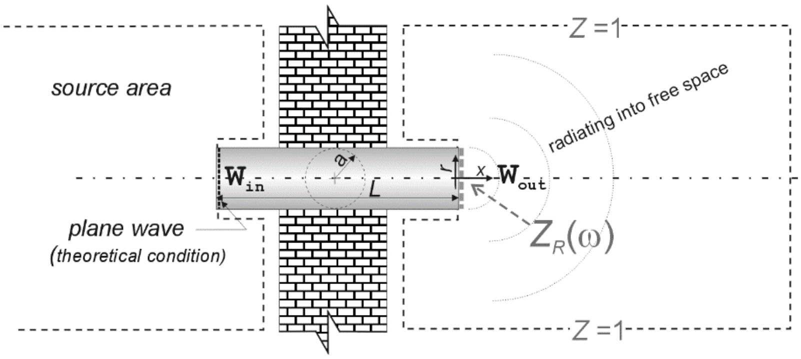

Figure 1. Cylindrical duct (pipe) links two environments: source area and free space

The difference between the sound energy on one side of the pipe and that radiated from the second side (both expressed in decibels) is called the sound transmission loss. The larger the sound transmission loss (in decibels), the smaller the amount of sound energy passing through and consequently, less noise heard. Transmission loss depends on frequency.

For the ideal case, when there is no ambient velocity and when the viscosity and thermal conduction are neglected, the energy density [1] in considered domain at a given time is described by

$$W = \frac{1}{2}\left(\rho v^2 + \frac{p^2}{\rho c^2}\right) \tag{1}$$

where ρ is density of fluid, v is the velocity of medium and c is the velocity of sound in fluid.

The first term in the above expression for energy density is recognized as the acoustic kinetic energy per unit volume, and the second term is identified as the potential energy per unit volume due to compression of the fluid.

Most of the energy entering and leaving the device is carried by plane waves. Hence, the sound energy incident and transmitted from the device can be found from the acoustic pressures associated with the incident and transmitted acoustic waves. For a plane wave, the kinetic and potential energies are the same. This yields the interpretation that the energy in a sound wave is moving in the direction of propagation with the sound speed [1].

A useful measure of the acoustic performance which depends only on the device is the frequency-dependent sound transmission loss (*TL*), which is defined as the ratio of the sound power incident on the device inlet to that of the sound power leaving the device at the outlet. Transmission loss is given by

$$TL = 10\log\frac{W_{in}}{W_{out}} \tag{2}$$

where W_{in} denotes the incoming power at the inlet of duct, W_{out} denotes the transmitted (outgoing) power at the outlet. The incoming and transmitted sound powers are given by equations

$$W_{in} = \int_S \frac{p_0^2}{2\rho c} dS,\, W_{out} = \int_S \frac{|p|^2}{2\rho c} dS, \tag{3}$$

where p_0 represents the applied pressure source amplitude and S - the area of the boundary where waves are incoming to the duct and outgoing from the duct. The incoming sound power is expressed by applied pressure source amplitude and not by pressure at the inlet. Calculated sound power at the inlet is the sum of sound power from the source, sound power reflected from pipe inlet, pipe outlet, and pipe boundaries.

One can derive average complex radiation impedance over the outlet of the cylindrical duct as integrate with cylindrical coordinates

$$Z_R = \frac{1}{S}\iint_S \frac{p}{v_n} dS = \frac{1}{\pi a^2}\int_0^{2\pi}\int_0^a r\left[\mathrm{Re}\left(\frac{p}{v_n}\right) + j\mathrm{Im}\left(\frac{p}{v_n}\right)\right] dr d\varphi, \tag{4}$$

where S is the area of the surface and $v_n = \mathbf{n}\cdot\mathbf{v}$ is the outward normal component of velocity on the surface. The outlet of the duct is a radiating surface where radiation impedance is a function of frequency and cylindrical coordinate r.

3. Pressure acoustic equations

The theory in this section is based on a handbook of acoustic [1]. A single wave equation for the acoustic part of the pressure for the case when the density varies with a position is

$$\frac{\partial^2 p}{\partial t^2} = \rho c^2 \nabla\cdot\left(\frac{1}{\rho}\nabla p\right). \tag{5}$$

An important special case is a time-harmonic wave, for which the pressure varies with time as

$$p(\mathbf{x}, t) = p(\mathbf{x})e^{i\omega t}, \tag{6}$$

where $\omega = 2\pi f$ is the angular frequency, with f denoting the frequency. In a time-harmonic case, the equation reduces to an inhomogeneous Helmholtz equation

$$-\frac{\omega^2 p}{\rho c^2} = \nabla \cdot \left(\frac{1}{\rho}\nabla p\right). \tag{7}$$

In this paper, for simplicity, the preferred work choice is to work in a non-dimensional frame of reference. Now some dimensionless variables will be introduced to make the system much easier to study [4]. Non-dimensionalized variables and scales are defined as follows: $\mathbf{x}' = \mathbf{x}/h$; $p' = p/\rho c^2$; $f' = hf/c$, where h is the characteristic distance (for instance pipe length L). Putting dimensionless variables into the time-harmonic wave equation we get

$$-\omega'^2 p' - \nabla'^2 p' = 0. \tag{8}$$

Since now primes will not be written and old variables symbols will be used.

4. Numerical results

In this section, the time-harmonic wave analysis of idealized cylindrical duct and outlet to free acoustic space is made. The frequency-dependent sound transmission loss of a hard-walled cylindrical duct pipe is calculated as well. The duct length is L and the radius is a Free acoustic space beyond duct's outlet (no reflected wave is coming back to duct) is modeled by a larger cylinder (length and radius equal $10a$) with unit impedance boundary condition. The axial symmetry of the geometry and the physics make it natural to set the model up in a 2D axisymmetric application mode.

All acoustic problems considered in this paper are governed by dimensionless equations with appropriate boundary and initial conditions and solved using standard computational code COMSOL Multiphysics. FEM calculations were made with second-order triangular Lagrange elements. Following boundary conditions for the dimensionless model were assumed: duct inlet - plane wave with amplitude, $p_0 = 1$; duct walls - sound hard boundary, $n \cdot (-\nabla p) = 0$; cylinder's boundaries - impedance boundary condition, $n \cdot (-\nabla p) = i\omega p/Z$, where $Z = 1$ is dimensionless impedance; axial symmetry - $r = 0$.

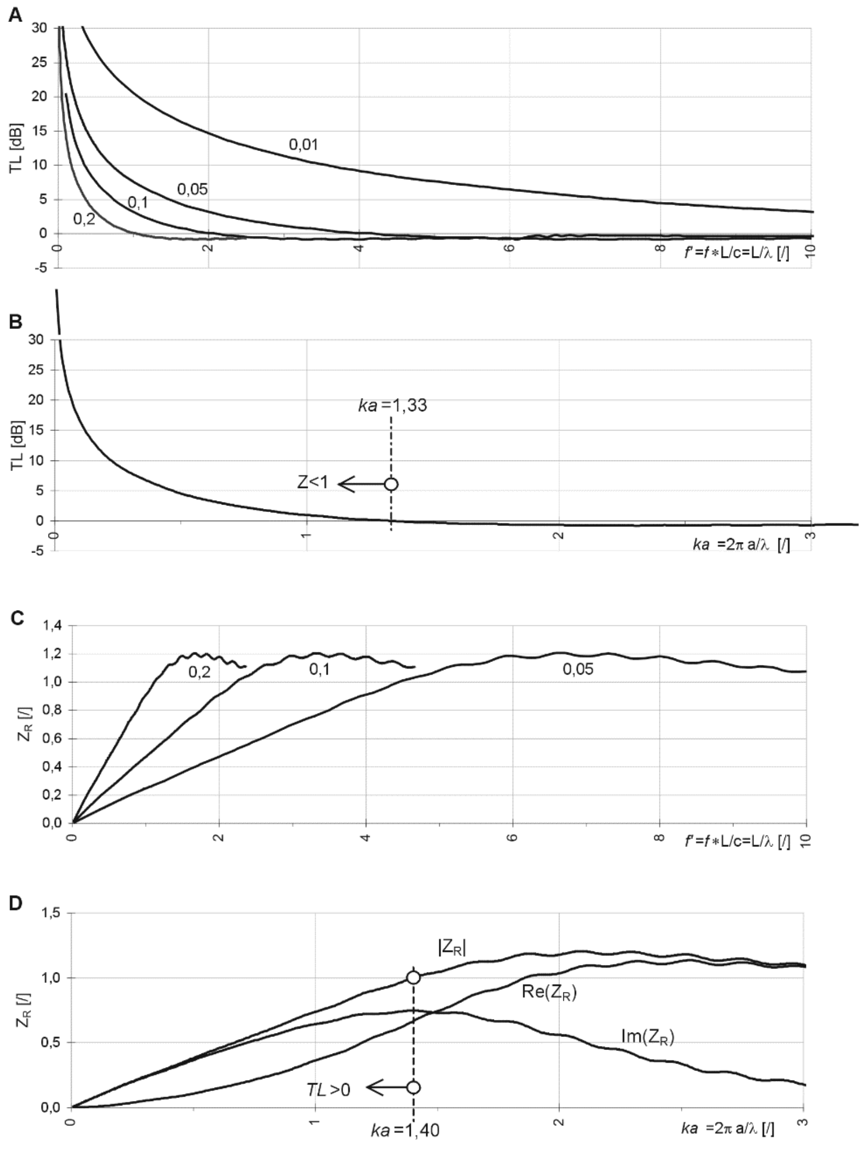

Figure 2. Comparison of numerical results of transmission loss (A, B) and radiation impedance in the cylindrical duct (C, D) in the function of dimensionless frequency (A, C) and diffraction parameter (B, D)

In this paper cylindrical ducts with different ratios (radius to length) equals 0.01, 0.05, 0.1, and 0.2 are analyzed. It is easy to observe (Figure 2A) that ducts with a smaller radius have greater possibilities to attenuate sound (value of TL is greater). As was mentioned earlier transmission loss and radiation impedance are frequency-dependent (Figure 2A, 2C). These quantities are the same for each cylindrical duct (with different ratio radius to length) when are considered as a function of diffraction parameter $k \cdot a$ (Figure 2B, 2D), where k is wave number.

5. Conclusions

In many papers, sound power transmitted from an outlet of device is calculated on the end of the device where impedance boundary condition or plane wave with zero pressure source amplitude is assumed. This assumption does not guarantee a proper model of the environment outside of the device outlet.

In this chapter numerical method for calculation of the frequency-dependent sound transmission loss of duct with an outlet to free acoustic space is presented. The frequency-dependent sound transmission loss and radiation impedance of the cylindrical duct with a given ratio radius to length were calculated as well.

All acoustic problems considered in this work are governed by dimensionless equations with appropriate boundary and initial conditions. Numerical results are obtained using standard computational code COMSOL Multiphysics using FEM. The FEM results calculated with a coarse mesh, show a good agreement with the experimental results.

There exists for each pipe critical frequency after which calculated and measured frequency-dependent sound transmission loss of duct begins to change in a different way [4]. If frequency grows then wavelengths diminish and become smaller than the size of finite element. Moreover, for high frequencies wave inside the hard-walled pipe is the superposition of longitudinal and transverse waves. These conclusions suggest that the computational FEM method with second-order triangular Lagrange elements is valid for only a limited range of frequencies or wavelengths. Beyond this range, it is necessary to use the ultraweak Helmholtz element.

Acknowledgments

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